Ex. 1.

1. $$-\frac{e^2}{r_0} = \frac{e^2}{r_0^3}(r_0^2+C)$$

$$-r_0^2 = r_0^2 + C \Rightarrow C = -2r_0^2$$

2. $$W = V(r) - V_0(r) = \frac{e^2}{r_0^3}(r^2+C) + \frac{e^2}{r} \quad \text{pour } 0 \le r \le r_0 \text{ , } 0 \text{ sinon}$$

3. $$E_{1s} = -R, \quad E_{2s} = E_{2p} = -\frac{R}{4} \quad E_{3s} = E_{3p} = E_{3d} = -\frac{R}{9}$$

4. $$\Delta E = \int_0^{r_0} r^2 R_{nl}^2(r) \left(\frac{e^2}{r_0^3}(r^2+C) + \frac{e^2}{r} \right)$$

$$\approx R_{nl}^2(r\approx 0) \cdot \left\{ \frac{e^2}{r_0^3} \int_0^{r_0} r^4 + Cr^2 \, dr + e^2 \int_0^{r_0} r \, dr \right\}$$

$$= R_{nl}^2(r\approx 0) \left\{ \frac{e^2}{r_0^3} \left\{ \frac{1}{5} r_0^5 + \frac{C}{3} r_0^3 \right\} + e^2 \frac{1}{2} r_0^2 \right\}$$

$$= R_{nl}^2(r=0) \left\{ \frac{e^2}{r_0^3} \left(\frac{1}{5} r_0^5 - \frac{2}{3} r_0^5 \right) + \frac{1}{2} e^2 r_0^2 \right\}$$

$$= R_{nl}^2(r=0) \left\{ \frac{e^2}{30} r_0^2 \right\} = R_{nl}^2 \cdot \alpha$$

$$\Delta E(1s) = \frac{e^2}{30} \cdot 4 \cdot \frac{r_0^2}{a_0^3} = \frac{2}{15} \frac{e^2}{a_0} \cdot \left(\frac{r_0}{a_0}\right)^2$$

$$\Delta E(2s) = \frac{e^2}{30} \cdot 4 \cdot \frac{r_0^2}{8a_0^3} = \frac{2}{15} \cdot \frac{e^2}{a_0} \left(\frac{r_0}{a_0}\right)^2 \cdot \frac{1}{8} = \frac{1}{60} \frac{e^2}{a_0} \left(\frac{r_0}{a_0}\right)^2$$

$$\Delta E(3s) = \frac{e^2}{30} \cdot 4 \cdot \frac{r_0^2}{27a_0^3} = \frac{2}{15} \frac{e^2}{a_0} \left(\frac{r_0}{a_0}\right)^2 \cdot \frac{1}{27} = \frac{2}{405} \frac{e^2}{a_0} \left(\frac{r}{a_0}\right)^2$$

$$\Delta E(2p) = \Delta E(3p) = \Delta E(3d) = 0$$

5. perturbation sonde présence de l'électron à $r \approx 0$
$= 0$ pour $l \neq 0$.
pour $l = 0$, pour $n \nearrow$, étendu de l'e^- plus grand
et pb. à l'origine plus faible : $\Delta E(1s) > \Delta E(2s) > \Delta E(3s)$

Ex 2.

1. a) $\vec{\ell}$: mom. angulaire de l'electron (orbital)
$\vec{s}$: mom. angulaire dû au spin de l'electron

Origin du terme s.o : interaction entre spin et ch. magnetique vu par l'electron grâce à son mvt dans le champs coulombienne

b) $H_{so} = \zeta \vec{\ell}.\vec{s} = \frac{\zeta}{2}(j^2 - \ell^2 - s^2)$

c) $\Delta E(1s)$: $\ell = 0$, $s = 1/2$, $j = 1/2$, $\Delta E(1s_{1/2}) = 0$

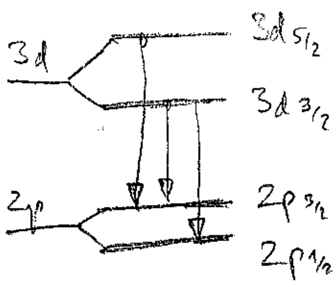
d) $\Delta E(2p)$: $\ell = 1$, $s = 1/2$, $j = 1/2$ $\quad \Delta E(2p_{1/2}) = \frac{\zeta\hbar^2}{2}\left(\frac{1}{2}\cdot\frac{3}{2} - 1\cdot 2 - \frac{1}{2}\cdot\frac{3}{2}\right) = -\zeta\hbar^2$

$j = 3/2$ $\quad \Delta E(2p_{3/2}) = \frac{\zeta\hbar^2}{2}\left(\frac{3}{2}\cdot\frac{5}{2} - 1\cdot 2 - \frac{1}{2}\cdot\frac{3}{2}\right) = \frac{\zeta}{2}\hbar^2$

e) $\Delta E(3d)$: $\ell = 2$, $s = 1/2$, $j = \frac{3}{2}$ $\quad \Delta E(3d_{3/2}) = \frac{\zeta\hbar^2}{2}\left(\frac{3}{2}\cdot\frac{5}{2} - 2\cdot 3 - \frac{1}{2}\cdot\frac{3}{2}\right) = -\frac{3}{2}\hbar^2$

$j = \frac{5}{2}$ $\quad \Delta E(3d_{5/2}) = \frac{\zeta\hbar^2}{2}\left(\frac{5}{2}\cdot\frac{7}{2} - 2\cdot 3 - \frac{1}{2}\cdot\frac{3}{2}\right) = \zeta\hbar^2$

f)



3 raies

2. a) $\Delta E_z(1s_{1/2}) = 2 \cdot \mu_B \cdot B \cdot m_j$, $m_j = \pm 1/2$

$\Delta E_z(2p_{1/2}) = \frac{2}{3}\mu_B \cdot B \cdot m_j$, $m_j = \pm 1/2$

$\Delta E_z(2p_{3/2}) = \frac{4}{3}\mu_B \cdot B \cdot m_j$, $m_j = \pm\frac{3}{2}, \pm\frac{1}{2}, 0$

b) nombre d'etats : $2j+1$

Ex 3.

a) $H_{hf} = \frac{A}{\hbar^2} \vec{I} \cdot \vec{J} = \frac{1}{2} \frac{A}{\hbar^2} \left(F^2 - I^2 - J^2 \right)$

b) etat fondamental : $l = 0$, donc $J = s = 1/2$

$F = |I - J|, \ldots I + J = 2, 3$

$\Delta E_{hf}(F=2) = -\frac{7}{4} A$, 5-fois deg.

$\Delta E_{hf}(F=3) = \frac{5}{4} A$, 7-fois deg.

fondamental : $|F=2, m_F\rangle$